

On the relationship between the size and surface coverage of starspots on magnetically active low-mass stars.

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ABSTRACT

We present a model that predicts the light curve amplitude distribution for an ensemble of low-mass magnetically active stars, under the assumptions that stellar spin axes are randomly orientated and that cool starspots have a characteristic scale length and are randomly distributed across the stellar surfaces. The model is compared with observational data for highly magnetically active M-dwarfs in the young cluster NGC 2516. We find that the best fitting starspot scale length is not constrained by these data alone, but requires assumptions about the overall starspot filling factor and starspot temperature. Assuming a spot coverage fraction of 0.4 ± 0.1 and a starspot to unspotted photosphere temperature ratio of 0.7 ± 0.05 , as suggested by the inflated radii of these stars compared to evolutionary model predictions and by TiO band measurements on other active cool stars of earlier spectral type, the best-fitting starspot angular scale length is 3.5^{+2}_{-1} degrees, or a linear scale length of $\sim 25\,000$ km. This linear scale length is similar to large sunspot groups, but 2–5 times smaller than the starspots recently deduced on an active G-dwarf using eclipse mapping by a transiting exoplanet. However, the best-fitting spot scale length in the NGC 2516 M-dwarfs increases with the assumed spot temperature ratio and with the inverse square root of the assumed spot filling factor. Hence the light curve amplitude distribution might equally well be described by these larger spot scale lengths if the spot filling factors are < 0.1 or the spot temperature ratio is > 0.9 .

Key words: stars: rotation – stars: magnetic activity; stars: low-mass – clusters and associations: NGC 2516.

1 INTRODUCTION

Starspots are a ubiquitous manifestation of magnetic activity in the photospheres of cool stars with convective envelopes. Their sizes, filling factors and temperatures are important constraints on the dynamo mechanism, which regenerates and amplifies the sub-photospheric magnetic field, and on the magneto-hydrodynamic processes which shape the emergence of magnetic fields from these sub-photospheric layers out into the photosphere and beyond (see reviews by Thomas & Weiss 2008; Strassmeier 2009). Beyond these diagnostic roles, starspots cause rotational modulation of light curves that enable stellar rotation periods to be estimated, are a nuisance source of radial velocity jitter when searching for exoplanets (e.g. Reiners et al. 2010; Barnes, Jeffers & Jones 2011), confuse the estimation of stellar radii in active, eclipsing binaries (e.g. Jeffers et al. 2006; Morales et al. 2010) and, if the filling factor is large, could significantly alter the structure of low-mass stars by blocking convective flux in their outer envelopes, leading to increased radii and decreased effective temperatures (e.g. Spruit & Weiss 1986; Chabrier, Gallardo & Baraffe 2007; MacDonald & Mullan 2012).

Jackson, Jeffries & Maxted (2009) estimated the average radii of fast-rotating late K- and M-dwarfs in the young open cluster

NGC 2516, by multiplying together their rotation periods and equatorial velocities. These magnetically active stars appear to have radii that are larger than both model predictions and the radii measured by interferometry for otherwise similar, magnetically inactive field stars. The radius discrepancy increases towards lower masses, reaching $\simeq 50$ per cent at a given luminosity in M4 stars. Jackson et al. interpreted this inflation in terms of a two-temperature photospheric model that required large (20 per cent to more than 50 per cent in the coolest stars) filling factors of dark starspots.

The dark starspot hypothesis was motivated by: (i) the qualitatively similar discrepancies seen in the magnetically active components of close, low-mass binary systems (Ribas et al. 2008; Morales et al. 2009; Torres, Andersen & Gimenez 2010), for which a similar explanation has been advanced (Chabrier et al. 2007; Morales et al. 2010); (ii) the similarity of the proposed spot filling factors and temperatures to those determined for very active G- and K-stars from careful modelling of their optical TiO absorption bands (filling factors of 20–50 per cent and temperature ratios between spots and unspotted photosphere of 0.65–0.76; O’Neal, Neff & Saar 1998; O’Neal et al. 2004; O’Neal 2006).

In addition to yielding rotation periods, the broadband light

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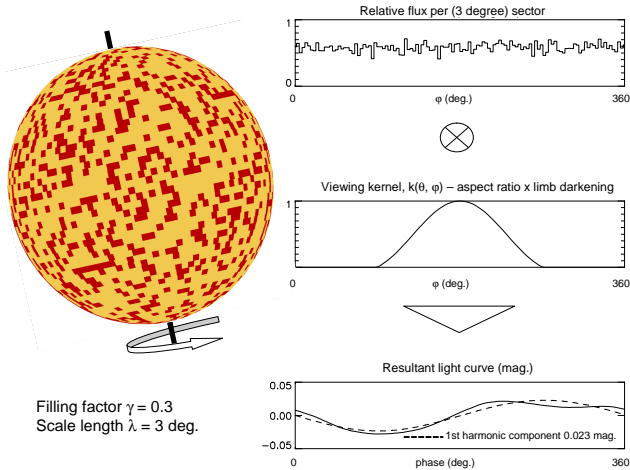


Figure 1. Simple model of a spotted star with a random covering of starspots of uniform area used to predict the probability density of light curve modulation amplitudes. The right hand panels illustrates how the flux from randomly distributed spots adds to produce the stellar light curve.

curves of magnetically active stars contain information about the distribution of spots on the stellar surface. Jackson et al. (2009) used rotation periods determined from I-band light curves that had typical first harmonic amplitudes of only 0.01–0.02 mag (from Irwin et al. 2007). Furthermore, Jackson & Jeffries (2012) showed that about half of the monitored low-mass members of NGC 2516, from the same data set, had no detectable light curve modulation at all, despite being just as magnetically active (judged by their chromospheric emission) as their periodic siblings and having a similar distribution of equatorial rotation velocities. It seems paradoxical to propose that such stars have large spot filling factors yet such small light curve amplitudes, but Jackson & Jeffries (2012) pointed out that the detectability of any rotational modulation might simply be governed by the degree of axisymmetry of the starspot distribution. The paradox might be resolved if the large filling factors were made up of many, randomly placed dark spots with typical angular diameters of ~ 2 degrees on the surface.

In this paper we place the scenario described by Jackson & Jeffries (2012) on a quantitative basis by presenting a simple numerical starspot model that predicts the light curve properties of an ensemble of active stars. We compare these model predictions with the observed properties of low-mass (M0–M4) stars in NGC 2516 and explore the relationship between spot filling factor, spot temperature, spot scale length and the distribution of light curve amplitudes. In section 2 we describe our model and its key assumptions; section 3 presents the model results and how well the model parameters are constrained by the observations; section 4 discusses the results in the context of the solar-stellar analogy and efforts to determine spot parameters using other techniques.

2 CALCULATION OF LIGHT CURVE MODULATION AMPLITUDES

In low-mass stars with a central radiative zone it is likely that an “ $\alpha\Omega$ ” dynamo is responsibly for amplifying magnetic field at the boundary between the radiative zone and convective envelope (Parker 1975). On the Sun this gives rise to latitude-dependent spot coverage and a strong latitude dependence has been predicted in fast-rotating stars with deep convective envelopes (such as the late-

K to M4 dwarfs considered here), such that starspots will emerge predominantly at high latitudes (Schüssler & Solanki 1992). In some cases this is confirmed by Doppler tomography of young, fast rotating K-stars which can have polar spots, but also some spots at lower latitudes (e.g. Stout-Batalha & Vogt 1999; Jeffers, Donati & Collier Cameron 2007). However, in other cases the spots on K-stars seem evenly distributed at all latitudes (Barnes et al. 2001). In early M-dwarfs there is no evidence for any strong latitudinal dependence of spot position from Doppler images (Barnes & Collier Cameron 2001). Some of our sample have spectral types cooler than M3.5, at which point the radiative core disappears and the nature of the dynamo may change to a turbulent “ α^2 ” dynamo (Chabrier & Küker 2006). There is little observational information on how spots might be distributed on the surface as a result. In what follows we will adopt the simplest assumption – that spots are randomly distributed on the surfaces of all our sample stars.

For simplicity, we also assume that any periodic light curve variations are due to dark starspots. In comparatively low activity stars like the Sun there is also a contribution from bright plages or faculae. Comparison of chromospheric and photospheric activity, suggests that the contribution of plages and faculae diminishes in more active stars (Radick et al. 1998; Lockwood et al. 2007).

Figure 1 illustrates the model used to predict the effects on the light curve amplitude produced by a random distribution of small starspots on the stellar surface. The surface of the star is divided into nominally equal cells of solid angle λ^2 , where λ is an angular scale length. Surface luminosities are randomly assigned to these cells according to the average starspot filling factor, γ and spot luminosity ratio, κ . When calculated for a set of stars, the resultant distribution of light curve amplitudes depends on four parameters:

- **Scale length**, λ is the angular distance between areas on the stellar surface that can show independent starspot activity. The absolute linear scale lengths for stars of different radii, R , can be compared by considering an angular size $\lambda R/R_\odot$, i.e. the equivalent angular distance on the surface of the sun. $\lambda = 2$ degrees corresponds to a cell covering 0.01 percent of the stellar surface. λ is not quite the same as the mean starspot size since the latter depends on how randomly distributed areas of starspot activity group together on the stellar surface which in turn depends on filling factor (see the example of a spotted star in Fig. 1).

- **Filling factor**, γ is the fractional area covered by starspots. A fraction $(1-\gamma)$ of the stellar surface is unaffected by starspot activity, with a surface flux corresponding to the effective temperature, T_o , of an unspotted star. The remaining fraction γ shows a reduced surface flux depending on the spot temperature. This spot temperature need not be uniform; spots may comprise an umbra, with a large temperature reduction, surrounded by a penumbra at intermediate temperatures.

- **Luminosity ratio**, κ is the ratio of the average surface flux of a spotted cell to that of an unspotted cell in the wavelength band of the measured light curve. In the simple case of a spot with a uniform temperature T_s , the κ value for the I-band light curves considered here declines as $(T_o/T_s)^n$, where $n \sim 5$ due to the usual Stefan’s law combined with the temperature dependence of the I-band bolometric correction. It turns out that κ cannot be constrained by the data we model and must therefore be assumed. The work of O’Neal et al. (1998), O’Neal et al. (2004) and O’Neal (2006) suggests that T_s/T_o lies in the range 0.65 to 0.76 for very active G- and K-dwarfs (that are somewhat warmer than our sample). Light curve modelling of M-dwarfs (e.g. Berdyugina 2005; Rockenfelder, Bailer-Jones & Mundt 2006) suggests that spots may only be a few hun-

dred Kelvin cooler than the unspotted photosphere, but these crude, single-spot models will greatly overestimate the spot temperature if there are many smaller spots. We consider a range of possibilities from $0.5 < T_s/T_o < 0.9$, corresponding to $0.03 < \kappa < 0.59$ for I -band light curves.

- **Completeness scale, σ .** The proportion of targets that will yield a *measured* rotation period as a function of light curve amplitude is characterised as a one-sided cumulative Gaussian distribution with a standard deviation of σ (in magnitudes), i.e. the completeness function varies from zero for small light curve amplitudes to unity for large light curve amplitudes. The value of σ is a free parameter in our model that depends on the observation cadence and sensitivity, which we will assume are uniform for a particular survey. However, within a survey, σ is likely to vary with target star brightness.

2.1 Calculation procedure

A Monte Carlo method is used to model the effects of randomly placed star spots on the the probability distribution of light curve amplitudes for a grid of scale lengths, filling factors and luminosity ratios. For each combination;

- The surface of the star is divided into cells of nominally equal solid angle of λ^2 steradians that are arranged in strips of constant latitude. A fraction γ of these cells are assigned a surface flux density of κ relative to the flux density of the unspotted surface. The selection of which cells are darker is made randomly.
- The flux densities from individual cells in each latitudinal strip are re-binned into longitudinal bins matching the longitudinal size of the original cells at the equator to give equal numbers of cells at all latitudes.
- The contribution from each of these re-binned cells are summed according to their area, viewing angle and limb darkening to give the stellar luminosity as a function of rotation phase.
- The variation of luminosity (relative to the mean) is analysed to determine the magnitude of the first harmonic component of light curve amplitude.
- Results of repeated simulations are accumulated to determine the probability distribution of light curve amplitudes as a function of λ , γ and κ .

To calculate the light curve the net flux density of each of the rebinned cells is first scaled according to the cell area (which varies as $\cos \theta$ where θ is the cell latitude) to give the net flux per cell. Fluxes from the cells in each latitudinal strip are then convolved with a viewing kernel, k and the result summed over all latitudes to give the light curve of the spotted star as a function of rotation, where;

$$k = \cos \theta \cos \phi (1 - \mu(1 - \cos \theta \cos \phi)) \quad \text{for } \frac{-\pi}{2} < \phi < \frac{\pi}{2} \quad (1)$$

where ϕ is the latitude, $\arccos(\cos \theta \cos \phi)$ is the viewing angle and the term $(1 - \mu(1 - \cos \theta \cos \phi))$ accounts for limb darkening. For the calculations in this paper a limb darkening coefficient of $\mu = 0.6$ is used (Claret, Diaz-Cordoves & Gimenez 1995), but the results are insensitive to this parameter.

In this model the number of cells in each latitudinal strip is rounded to the nearest integer. Hence the solid angle of the cells is not exactly λ^2 and the number of cells is not exactly $4\pi/\lambda^2$. The fractional error in the average cell solid angle introduced by this approximation varies with scale length from $\simeq 0.5$ percent for $\lambda = 0.01$ radians to $\simeq 2$ percent for $\lambda = 0.5$ radians. This level

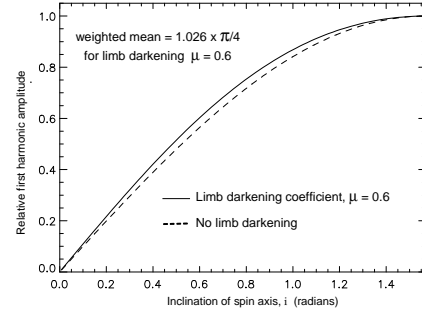


Figure 2. Variation of the light curve modulation amplitude with inclination of the stellar spin axis.

of error is negligible in the context of this paper when compared to the much larger uncertainties in the fits to measured data.

The calculation procedure above is valid for a star at inclination $i = 90$ deg. The modulation amplitude for randomly spotted stars viewed at other inclinations is attenuated by a factor that approximates to $\sin i$. To calculate the attenuation factor taking account of limb darkening it is sufficient to evaluate the attenuation factor for a single spot as a function of inclination and spot latitude which can then be averaged over the stellar surface. Consider a spot at coordinates (θ, ϕ) on the surface of a star of unit radius and spin axis inclination i . Viewed in Cartesian co-ordinates, with z measured along the line towards the observer, a point x, y, z is visible if $z > 0$, where

$$\begin{aligned} x &= \sin i \sin \theta - \cos i \cos \theta \cos \phi, \\ y &= \cos \theta \sin \phi, \\ z &= \cos i \sin \theta + \sin i \cos \theta \cos \phi. \end{aligned} \quad (2)$$

Taking account of limb darkening, the relative luminosity of the spot is given by $\cos q(1 - \mu(1 - \cos q))$ where q is the angle of the normal at point (x, y) to the line of sight and hence the relative contribution of a unit area starspot to the light curve amplitude is given by

$$\rho_i(\theta) = 2 \int \cos q(1 - \mu(1 - \cos q)) \cos \phi \, d\phi, \quad (3)$$

where $q = \arcsin(\sqrt{x^2 + y^2})$ and $\cos \phi > -\tan \theta / \tan i$ (i.e. $z > 0$). In our model, the spots are uniformly distributed over the stellar surface, hence their probability density varies as $\cos \theta$, giving a weighted average of

$$\bar{\rho}_i = 2 \int \rho_i(\theta) \cos \theta \, d\theta \quad (4)$$

Figure 2 shows a plot of $\bar{\rho}_i / \bar{\rho}_{\pi/2}$ as a function of inclination. For randomly distributed spots and $\mu = 0.6$, this angular function is within a few per cent of $\sin i$, with a weighted mean of $1.026\pi/4$. To a good approximation, the light curve amplitudes of stars viewed at right angles to the line of sight can be scaled by this mean value to give the distribution of light curve amplitudes averaged over all viewing angles.

An implicit assumption here is that the spin axes of the stars are randomly oriented in space. The validity of this assumption was discussed in detail by Jackson & Jeffries (2010a) for collections of stars, similar to those discussed here, in the young Pleiades and Alpha Persei clusters. Whilst the assumption of randomness is difficult to confirm, there is certainly no evidence for any strong intrinsic

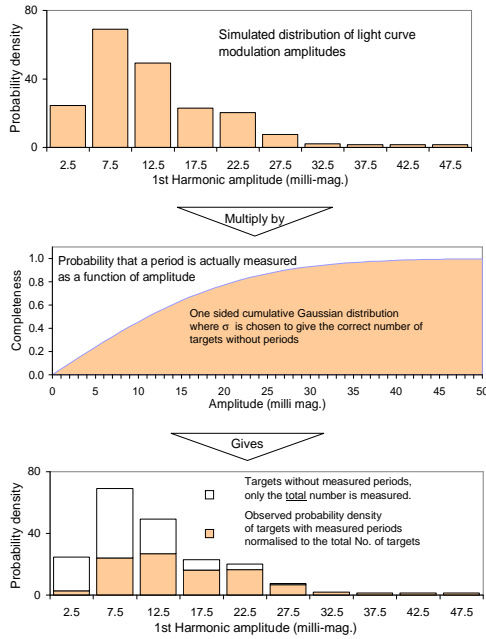


Figure 3. The process used to scale the simulated values of the probability density of light curve amplitudes to give a probability density function for stars with measured periods that can be compared with observations.

sic alignment. As we are considering a complete sample of stars in NGC 2516, whether they exhibit periodic modulation or not, we do not expect any observational selection bias in the inclination angles either (see Jackson & Jeffries 2012). The effect of any alignment would be to alter the predicted light curve amplitudes by a factor given by the y-axis values in Fig. 2 divided by the average value of $1.026\pi/4$. In the absence of any evidence for spin axis alignment in NGC 2516 or any other cluster, we do not consider this further.

Finally, the model light curve amplitude distribution is multiplied by the previously defined completeness function, which is the probability that a period will be measured for a given light curve amplitude (see Fig. 3). This gives the probability density of *measured* light curve amplitudes normalised to the total number targets. It is this latter probability density that can be compared directly with observational data.

2.2 Variation of light curve modulation amplitudes with scale length and filling factor.

Figure 4 summarises the properties of model light curve amplitude distributions predicted for sets of stars with a random distribution of starspots. These are the full amplitude distributions before scaling by any completeness function. The left hand plot shows the predicted probability density of modulation amplitudes for stars with $\gamma = 0.3$, $\kappa = 0.16$ ($T_s/T_o = 0.7$ for uniform spots) and for $\lambda = 2, 4$ and 8 degrees. The right hand plot shows the variation of mean amplitude for a wide range of λ and γ . These results indicate that to achieve the small mean amplitudes characteristic of the active low mass stars considered here (typically 0.015 mag, see section 3), together with high spot filling factors, requires small values of scale length. For example a filling factor of $\gamma = 0.3$ would correspond to $\lambda \simeq 3$ degrees. For these small scale lengths the mean amplitude, for a fixed value of κ , scales roughly as $\lambda\gamma^{1/2}$.

3 CONSTRAINING STARSPOT SIZE USING MEASURED LIGHT CURVE AMPLITUDES

In this section, model results are compared to measured light curve amplitude distributions for a sample of active low mass stars in NGC 2516. In principle we would like to constrain the four parameters, scale length, λ , filling factor γ , luminosity ratio, κ and completeness scale σ . It turns out that λ and σ can be determined from fitting the model to the measured amplitude distribution but the results depend to some extent on γ and κ , which must be estimated from other observations of the target population or more general considerations.

3.1 Measured distributions of light curve amplitudes

NGC 2516 is a young (150 Myr) open cluster with a population of low mass stars ($0.2 \leq M/M_\odot \leq 0.7$) approaching or on the zero age main sequence. *I*-band light curve amplitudes and rotational periods were measured for a large sample of candidate members by Irwin et al. (2007). A spectroscopic survey was used by Jackson & Jeffries (2010b) and Jackson & Jeffries (2012) to confirm membership for 210 stars with rotation periods and 144 stars where no period was found. In these papers it was shown that there were no significant differences in the colour-magnitude diagrams, the projected equatorial velocity distributions or the levels of chromospheric magnetic activity for these two subsets.

Figure 5 shows the first harmonic light curve amplitude distributions (for the stars with measured periods) and summarises the mass range, fraction of stars with a measured period and the mean first harmonic light curve amplitudes (all taken from Jackson & Jeffries 2012) for the low-mass stars in this sample ($M_I > 7.3$, $M < 0.57 M_\odot$). All these stars, and also the stars without measured periods in this magnitude range, show saturated levels of chromospheric activity. This is important, because it means we do not expect the filling factor of spots to vary with rotation rate and can treat the sample as a single population. About 40 per cent of the sample have $M < 0.35 M_\odot$ and may be fully convective, according to a theoretical mass-magnitude relationship from Baraffe et al. (2002). The results are shown in four equal bins of absolute *I* magnitude. The proportion of stars with and without measured periods in each bin are true estimates, corrected for any bias due to the preferential targeting of stars with measured periods in the spectroscopic sample (see Table 5 of Jackson & Jeffries 2012). The form of the amplitude distributions mirrors the model distributions shown in Fig. 3, with an initial increase in frequency (moderated by the completeness function described in section 2), a peak in the range 0.01 to 0.02 mag followed by a rapid decay, with no measured amplitudes > 0.05 mag. The error bars on the measured data represent Poissonian uncertainties.

3.2 Constraining scale length and completeness scale

A maximum likelihood technique is used to constrain possible values of λ and completeness scale σ , as a function of γ and κ . To this end a grid of probability densities for the light curve amplitudes is generated in 0.005 magnitude bins from 0 to 0.1 mag for $\lambda = 1$ to $100\sqrt{\gamma}$ degrees and $\sigma = 0$ to 0.1 for a series of values of γ and κ (see below). As there are less than 20 stars in each bin, chi-squared methods would yield biased results, so the most probable fit is found using the modified form of the Cash statistic (Cash 1979).

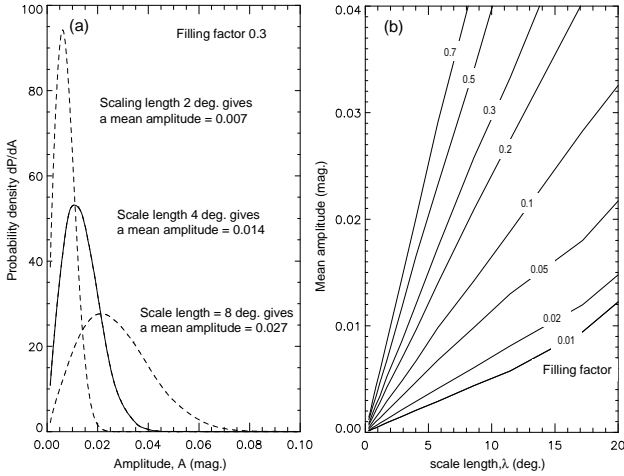


Figure 4. Monte Carlo simulations of light curve amplitudes for stars with a large number of randomly distributed starspots of uniform area where the spotted areas have a luminosity ratio of $\kappa = 0.16$ relative to the unspotted photosphere. Plot (a) shows the probability density for stars with a filling factor $\gamma = 0.3$ for various spot scale lengths. Plot (b) shows the mean light curve amplitude as a function of scale length for various filling factors.

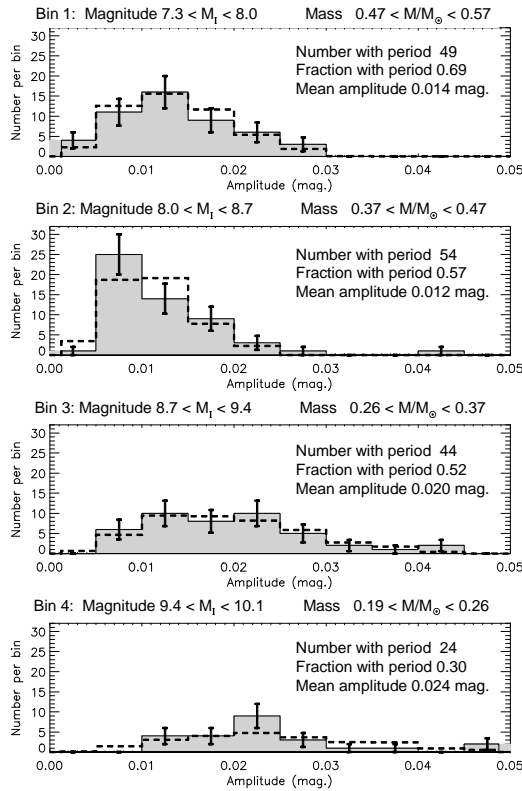


Figure 5. The probability density of light curve modulation amplitudes for low mass stars in NGC 2516 (Jackson et al. 2012). Results are shown in four bins of absolute I magnitude with equivalent masses from the models of Baraffe et al. (1998 & 2002). The shaded histogram shows the number of stars with measured periods as a function of first harmonic amplitude, together with their expected Poisson uncertainties. The dashed lines show numbers predicted from a Monte Carlo simulation of a randomly spotted star using “best fit” values for λ and σ (see section 3.2), assuming $\gamma = 0.3$ and $\kappa = 0.16$.

$$C = 2 \sum_{i=1}^N [y(x_i) - y_i + y_i (\ln y_i - \ln y(x_i))] \quad (5)$$

where y_i are the observed data and $y(x_i)$ are the model values. This form of the Cash statistic is appropriate for binned data with small counts and can be treated in the same way as χ^2 to determine goodness of fit and determine the confidence levels of fitted parameters.

Figure 6 shows contour plots of the modified Cash statistic constructed by modelling each of the data sets in Fig. 5. The vertical axis shows the completeness scale, σ and the horizontal axis is the scale length, λ . Contours mark 95 per cent confidence limits around the the combination of σ and λ that best fit the observations. Results are shown for four possible values of filling factor, $\gamma = 0.05, 0.1, 0.3, 0.5$ at a fixed luminosity ratio of $\kappa = 0.16$ (equivalent to $T_s/T_0 = 0.7$ for uniform spots). Notes on each plot show the minimum value of the Cash statistic for each case and the average value of the resultant probability that the best fit model is a good fit to the measured distribution. Examples of the model predictions are shown in Fig. 5 for the case of $\gamma = 0.3$ and $\kappa = 0.16$. It is important to note (see the C_{\min} values in Fig. 6) that all the combinations of γ and κ that we tested yield statistically acceptable fits with an appropriate choice of λ and σ . That is, these observations alone are incapable of constraining γ or κ .

Figure 6 shows that the value of σ is reasonably independent of γ . However, σ does vary, as expected, with absolute magnitude, increasing from 0.01 to 0.025 mag over the first three subsets and rising sharply to 0.06 mag for the faintest subset. This corresponds to completeness values of 0.97, 0.93, 0.66 and 0.33 for light curves of amplitude 0.02 mag. This is reasonably consistent with the expected variation in measured period completeness with magnitude for stars in the Irwin et al. (2007) survey from which the data were taken.

The most likely value of λ varies as $\simeq \gamma^{-1/2}$ and also perhaps weakly with absolute I magnitude. Figure 7 shows a more detailed plot of the variation of the best fitting λ as γ is varied. The uncertainties shown on this plot correspond to 68 per cent confidence intervals in one parameter.

Finally, Fig. 8 summarises the results obtained for different values of κ ($0.03 \leq \kappa \leq 0.59$, corresponding to $0.5 \leq T_s/T_0 < 0.9$ for uniform spots). The error bars here indicate the largest and smallest values of λ obtained from the four M_I subsamples in Fig. 5. This plot shows that for a given filling factor, the best-fitting value of λ increases with κ and is roughly proportional to $(1 - \kappa)^{-1}$.

4 DISCUSSION

The main motivation for these simulations was the suggestion by Jackson & Jeffries (2012) that the small light curve amplitudes seen in the young, active low-mass stars of NGC 2516 were compatible with large spot coverage fractions, and might constrain the typical spot size. The results shown in Fig. 8 confirm this idea. Both the small observed light curve amplitudes of the periodic stars and the fraction of cluster members for which rotation periods could not be found can be explained by a random distribution of small, dark spots on the stellar surface. Jackson et al. (2009) estimated that 50 per cent or more surface coverage by dark spots ($T_s/T_o = 0.7$) may be required in the coolest stars of this sample to explain their large radii compared with standard evolutionary models. Fig. 8

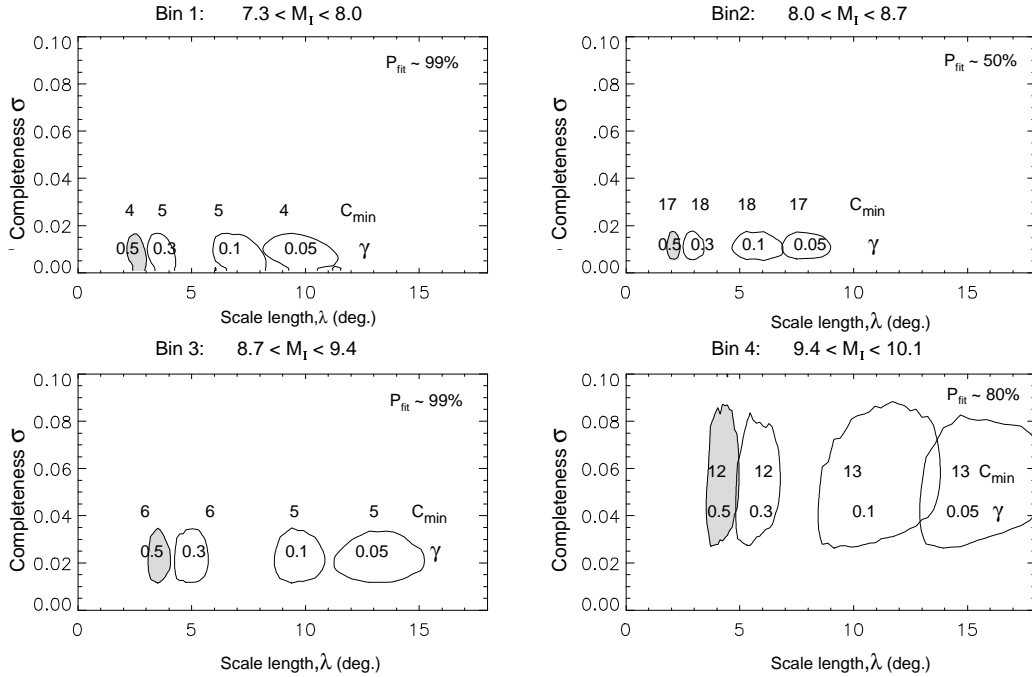


Figure 6. Contour plots of the Cash statistic as a function of scale length, λ and completeness function σ . Contours indicate the 95 per cent interval around the combination of λ and σ that best fits the four subsets of measured data in Fig. 5. Results are shown for a luminosity ratio $\kappa = 0.16$ at four values of filling factor, γ . Also shown are the minimum value of the Cash statistic for each case and the average probability of fit to each subset of measured data.

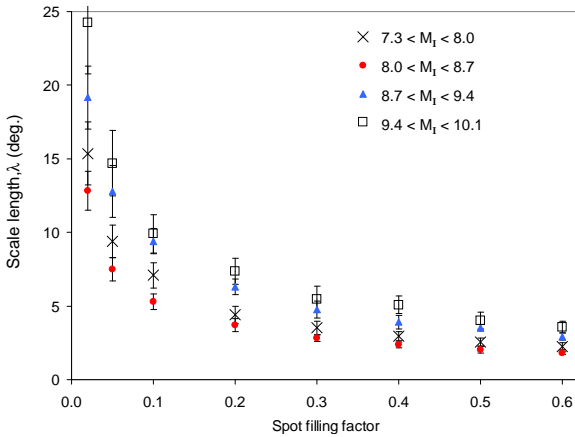


Figure 7. The variation of the best-fitting scale length as a function of with spot filling factor, for a spot luminosity ratio of $\kappa = 0.16$. The results are shown for the data in four subsamples of absolute I-magnitude (see Fig. 5). The error bars indicate 68 per cent confidence intervals.

shows that such spot coverage leads to inferred spot scale lengths of 3 ± 1 degrees.

The estimated characteristic spot scale length depends on (a) the assumed spot coverage fraction and spot temperature and (b) the assumption of a random spot distribution. Neither of these can be independently constrained by just the light curve data. Fig. 8 demonstrates that the best-fitting spot scale length could be significantly larger if the spot coverage were smaller or if the spots were hotter. The light curve amplitude distributions alone could equally well be explained by ~ 20 randomly placed spots of diameter 10 degrees ($\gamma = 0.05$) or ~ 2500 spots of diameter 3 degrees ($\gamma = 0.5$).

The indirect estimate of spot coverage and temperature from Jackson et al. (2009) assumed that the inflated radii observed for these magnetically active stars are solely due to starspots. This followed empirical evidence that larger radii are also seen in the low-mass components of fast-rotating, eclipsing binary stars and that this radius discrepancy has been linked with magnetic activity (López-Morales 2007). Chabrier et al. (2007) showed that 30–50 per cent coverage by black spots could reproduce these results too. However, other effects, such as a reduction of convective efficiency or inhibition of the onset of convection by interior magnetic fields might also increase radii and thus reduce the required spot coverage (see also MacDonald & Mullan 2012). Hence these indirect estimates of γ are possibly upper limits.

Independent determinations of the spot coverage in active stars depend crucially on the technique used. Analyses of photometric light curves or Doppler imaging maps probably underestimate total spot coverage because of their limited spatial resolution or lack of sensitivity to axisymmetric spot distributions (see Solanki & Unruh 2004). The most direct estimates of γ and T_s/T_0 are likely to come from measuring a number of TiO absorption bands in high resolution spectra and fitting them with two-temperature models, using the spectra of magnetically inactive stars as templates (see O’Neal et al. 1998). Results are reported for a number of very active G- and K-type stars by O’Neal et al. (2004) and O’Neal (2006). These include three active young dwarf stars, EK Dra (G1.5V), LQ Hya (K0V) and EQ Vir (K5V), for which $\gamma = 0.4 \pm 0.1$ and $T_s/T_0 = 0.70 \pm 0.05$ were determined. Unfortunately, the same technique is ineffective for M-dwarfs since their unspotted photospheres also show strong TiO absorption bands (O’Neal et al. 2005).

If we were to extrapolate and assume that similar parameters ($\gamma = 0.4 \pm 0.1$, $T_s/T_0 = 0.70 \pm 0.05$) apply to the active M-dwarfs of NGC 2516, then a scale length $\lambda = 3.5^{+2}_{-1}$ degrees is implied

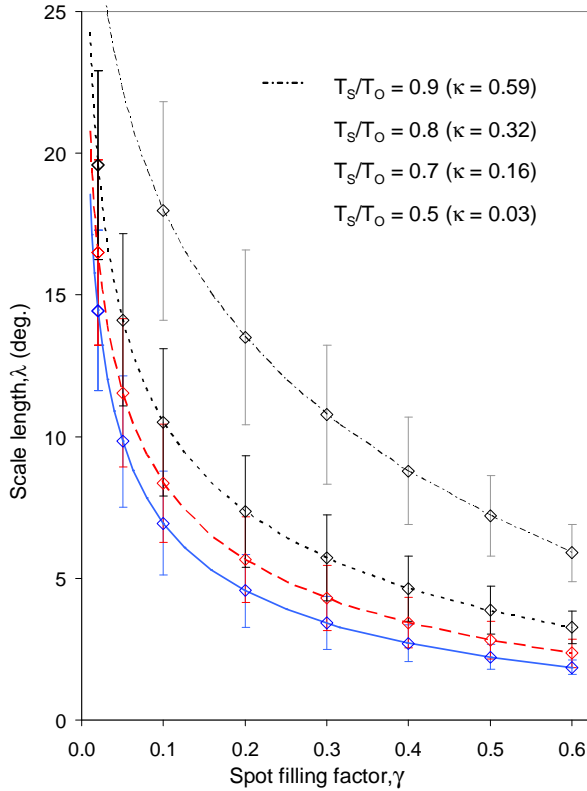


Figure 8. Possible constraints on the scale length for independent areas of starspot activity as a function of filling factor, γ and temperature ratio T_s/T_o . Lines indicate the mean value of λ that best fits measured data shown in Fig. 5. The error bars indicate the highest and lowest λ values obtained from fitting the data in each of the four absolute I-magnitude sub-samples in Fig. 5.

(see Fig. 8). The legitimacy of this extrapolation could be questioned on the basis that the stars studied by O’Neal et al. were of earlier spectral type, with shallower convection zones. As discussed in section 2, it is possible that the nature of the dynamo changes as the convection zone deepens and especially when stars become fully convective, which may be the case for the coolest 40 per cent of stars (roughly the third and fourth bins in Fig. 5) in our sample. However, the vast majority of our sample are M0–M4 dwarfs for which there is reasonable evidence that magnetic activity is generated and manifested in a similar way to K-dwarfs. This includes the similarity of rotation-activity relationships between G-, K- and M-dwarfs as cool as type M4 (Jeffries et al. 2011; Reiners, Joshi & Goldman 2012) and the similar spot filling factors and distributions in active K- and early M-dwarfs inferred from Doppler tomography (Barnes & Collier Cameron 2001 and see below). Counter to this, there is some evidence from Zeeman Doppler imaging that the large scale magnetic field does undergo a change towards a more axisymmetric, poloidal topology at the fully convective boundary (beyond type M3; Morin et al. 2008, 2010). How this relates to photospheric fields at the scale of starspots is unknown, although the bulk of magnetic energy still appears to reside at smaller size scales (Reiners & Basri 2009). In summary, it is possible our extrapolation is invalid for the lowest mass stars of our sample.

The assumption of random spot coverage is the simplest approach we could have adopted, but as discussed in section 2, there

is some evidence that real low-mass stars may behave differently. Doppler images have revealed long-lived spots, or unresolved spot groups, at high latitudes or covering the rotational poles for some young K-dwarf stars at some epochs (e.g. on the rapidly rotating K-type ZAMS stars AB Dor in 1993/1994, Jeffries et al. 2007 and BO Mic in 2002, Barnes 2005), but not others (e.g. AB Dor in 1989, Kürster, Schmitt & Cutispoto 1994; BO Mic in 1998, Barnes et al. 2001). A concentration of spots towards high latitudes would reduce light curve amplitudes for a given γ , so larger spot scale lengths would be required to compensate. However, the effect would not be large; even if half the spot coverage were concentrated in an axisymmetric polar cap this would only increase the required λ by a factor of $\sqrt{2}$. In any case, the situation for fast-rotating M-dwarfs may be different. Doppler images of HK Aqr and EY Dra, M1-2 dwarfs with rotation periods < 1 day, reveal spots either at low latitudes or with no clear latitude dependence at all (Barnes & Collier Cameron 2001). Longitudinal asymmetries or preferential spot longitudes are more difficult to assess. For example if the presence of a spot or spot group at one longitude made it more likely that further spots would emerge at similar longitudes then this would increase photometric modulation for a given γ and alter the relationship between λ and γ in Fig. 8. Any attempt to observationally identify “active longitudes” in single stars is hampered by the possibility of differential rotation and a lack of spatial resolution.

The spot scale length implied by our simple model can be converted to a linear scale length if the stellar radius is known. Jackson et al (2009) estimated radii of 0.4–0.6 R_\odot for the stars in NGC 2516 considered here, which for $\lambda \simeq 3.5$ degrees implies absolute scale lengths of order 25 000 km. Baumann & Solanki (2005) have studied the distribution of spot sizes on the Sun, finding that both the sizes of individual spots and of spot groups are well-represented by log-normal distributions. A starspot area of $6 \times 10^{14} \text{ m}^2$ is a factor of 5–6 larger than the modal area (umbra plus penumbra) of individual sunspots ($\simeq 1$ degree diameter), but only a factor of 2–3 larger than a typical sunspot group and well within the observed dispersion.

High cadence, high signal-to-noise ratio photometry is now capable of estimating the sizes of individual starspots or starspot groups in systems where the spots are occulted by an exoplanet. Wolter et al. (2009) and Silva-Valio et al. (2010, 2011) have analysed modulation of the light curve during exoplanetary transits of a reasonably rapidly rotating ($P=4.5$ days) active G7V host, CoRoT-2. They found that typically the exoplanet transits 5 spots as it crosses the stellar disc and that these spots (or spot groups) have a diameter of 40 000 to 150 000 km with temperatures in the range 3600 – 5000 K (for $T_{eff} = 5625 \text{ K}$) giving $0.6 < T_s/T_o < 0.9$ and that the transited (low latitude) stellar region has a 10–20 per cent spot coverage. At these relatively low filling factors, the “spot size” is a roughly equivalent parameter to the scale length considered in this paper. Bearing in mind that the quality of the CoRoT data limited the analysis to spot sizes larger than 30 000 km and since there is some degeneracy between spot size and spot temperature in both their and our analyses, it seems that these spots may be only a little larger than those we have deduced for the very active M-dwarfs in NGC 2516. Alternatively, we could reverse this chain of argument and say that if the spots on the NGC 2516 stars were of a similar absolute size and temperature ratio to those of CoRoT-2, and randomly distributed over the stellar surface then from Fig. 8, the spot filling factor would be $\gamma < 0.1$. Alternatively, for a spot coverage of $\gamma \simeq 0.4$ then T_s/T_o would need to be > 0.9 .

In summary this paper shows that both the small light curve amplitudes observed in a set of fast-rotating, young, magnetically active M-dwarfs, and the lack of rotational modulation seen in a large fraction of their siblings, *could* be explained by a starspot model consisting of large filling factors of dark spots that are randomly distributed on the stellar surface. If the M-dwarfs in NGC 2516 have a spot coverage fraction $\gamma \sim 0.4 \pm 0.1$ and a spot/photosphere temperature ratio of $T_s/T_0 \sim 0.7 \pm 0.05$, as suggested by extrapolation of the TiO modelling of very active K dwarfs (O’Neal et al. 2004, 2006), then the scale length between independent areas of starspot activity is $\lambda \simeq 3.5^{+2}_{-1}$ degrees (or 25 000 km). This scale length varies as $\gamma^{-1/2}$ and increases with the assumed spot temperature, neither of which can be constrained by the light curve data. There is an urgent need to independently determine these parameters in lower-mass active stars, both to address the issue of typical spot sizes and also to assess the possible influence of spots in inflating stellar radii above the predictions of current evolutionary models. The spot scale lengths found above are only a little larger than typical sunspot groups but a little smaller than the spot sizes so far inferred from mapping using transiting exoplanets. If these small spot scale lengths are confirmed then this complicates the interpretation of Doppler and Zeeman Doppler imaging maps, where typical angular resolutions are in the range 3 – 10 degrees.

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